

## Second Congress of Greek Mathematicians

SCGM–2022, Athens, July 4-8, 2022

### Session Algebra, Number Theory and Combinatorics

#### Programme

	Τετάρτη	Πέμπτη	Παρασκευή
9:00-10:00	H. Geranios	V. Gazaki	T. Moulinos
10:15-10:45	N. Tsakanikas	D. Stergiopoulou	A. Terezakis
11:00-11:30	coffee break	coffee break	coffee break
11:30-12:30	κεντρική	κεντρική	κεντρική
12:30-13:30	κεντρική	κεντρική	κεντρική
13:30-15:00	διάλειμμα	διάλειμμα	διάλειμμα
15:00-15:30	κεντρική	κεντρική	I. Kaperonis
15:30-16:00			I. Tsouknidas
16:00-16:30	coffee break	coffee break	coffee break
16:30-17:00	V. Petrotou	M. Savvas	A. Hayash
17:00-17:30	S. Zikas		L. Vaso
17:30-18:00	M. Melistas	G. Dalezios	διάλειμμα
18:00-18:30	διάλειμμα	διάλειμμα	D. Tsipa
18:30-19:00	D. Deligeorgaki	E. Chavli	S. Afentoulidis
19:00-19:30	V. Moustakas	K. Karagiannis	
19:30-20:00	C. Tatakis	K. Psaromiligkos	

## Abstracts & Titles

**Spyros Afentoulidis (Université de Lorraine, Metz)**

### **Algebraic Dirac operators and Dirac cohomology**

In this talk, we will present the basic theory of the algebraic Dirac operators in the context of Lie algebra Representation Theory. More precisely, if  $\mathfrak{g}$  is a finite dimensional complex simple Lie algebra with Cartan decomposition  $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{p}$  and universal enveloping algebra  $U(\mathfrak{g})$ , while  $\mathbf{C}(\mathfrak{p})$  is the Clifford algebra associated to the vector space  $\mathfrak{p}$ , we define the corresponding Dirac operator  $D$  to be some  $K$ -invariant element of the algebra  $U(\mathfrak{g}) \otimes \mathbf{C}(\mathfrak{p})$ .

If  $X$  is a  $U(\mathfrak{g})$ -module and  $S$  is a spin module for  $\mathbf{C}(\mathfrak{p})$ , then  $D$  acts on the tensor product  $X \otimes S$ , this action commutes with the action of the Lie algebra  $\mathfrak{k}$  on  $X \otimes S$  while it encodes crucial information for  $X$ . Once we present thoroughly the above construction, we will discuss fundamental results of this theory while we will try to present some recent results.

**Eirini Chavli (University of Stuttgart)**

### **Nakayama algebras and 321-avoiding permutations**

Nakayama algebras appear in modular representation theory of groups and they are defined as a quotient of the algebra of upper triangular matrices. In this talk we will explain the connection of these algebras with a combinatorial object: the 321-avoiding permutations. We will also associate the properties of these permutations with the representation theory and homological algebra of Nakayama algebras (joint work with Rene Marczinzik).

**Georgios Dalezios (National and Kapodistrian University of Athens)**

### **Reedy categories and finite dimensional algebras**

Reedy categories form a generalization of the category of finite ordinals and monotone maps between them, which is of fundamental importance in algebraic topology. In this talk, we will present a linear version of Reedy categories and their relation with certain finite dimensional algebras, more precisely, with quasi-hereditary algebras. In addition, we will investigate the construction of certain model categories on functor categories with source linear Reedy categories, obtaining a linear analogue of a classic result of Kan. This is based on joint work with Jan Stovicek.

**Danai Deligeorgaki (KTH)**

### **Gorenstein decomposable models**

Discrete decomposable models are of wide use throughout statistics and data science. An advantage of these models is that they are associated to decomposable simplicial complexes, a family of complexes that facilitate probabilistic inference methods including variable elimination. In this talk, we are interested in (the combinatorics of) the toric varieties, and their corresponding polytopes, arising from discrete decomposable models. In particular, we provide a characterization of discrete decomposable models whose associated toric variety is Gorenstein.

**Valia Gazaki (University of Virginia)**

**Torsion phenomena for zero-cycles on a product of curves over a number field**

Let  $X$  be a smooth projective variety over a field  $k$ . The Chow group of zero-cycles on  $X$  is a generalization of the Picard group of a curve. Similarly to the case of curves, there is an Abel-Jacobi map from the Chow group to the  $k$ -rational points of an abelian variety, analogous to the Jacobian of a curve. When  $X$  is a curve, the Abel-Jacobi is essentially an isomorphism. In higher dimensions however it can have a nontrivial kernel. When  $k$  is a finite extension of the rational numbers, a famous conjecture of Bloch and Beilinson predicts that the kernel of the Abel-Jacobi is a torsion group. However, outside the world of curves, there is hardly any evidence for this conjecture. In this talk I will focus on the case when  $X$  is a product of smooth projective curves and construct infinitely many nontrivial examples that satisfy a weaker form of the Bloch-Beilinson conjecture. This relies on a recent joint preprint with Jonathan Love.

**Haralampos Geranios (University of York)**

**On self-extensions of irreducible modules over symmetric groups**

This talk is on the modular representation theory of the symmetric groups. An important and longstanding conjecture in this area suggests that there are no (non-trivial) self-extensions of irreducible modules for symmetric groups over fields of odd characteristic. We will talk about some recent developments on this topic and highlight several new positive results on this conjecture. The talk is based on a joint work with S. Kleshchev and L. Morotti.

**Andreas Hayash (University of Massachusetts Amherst)**

**Quiver Zastava spaces and quantum groups**

Zastava spaces are certain algebraic varieties associated to a semisimple algebraic group that have numerous applications to geometric representation theory. Recently, the construction of these spaces was generalized by Mirkovic, Yang, and Zhao to associate a Zastava space to a pair consisting of a graph together with a quadratic form on the free abelian group generated by its vertices. In this talk, I will give an overview of their construction via factorizable line bundles on configuration spaces, as well as current work in progress relating sheaves on the Zastava spaces to quantum groups.

**Ilias Kaperonis (National and Kapodistrian University of Athens)**

**K-Absolutely pure complexes**

Projective and injective resolutions of modules play an important role in the study of Homological Algebra. However we can view the category of modules over a ring as a subcategory of the complexes of the ring. Spaltestein in 1972 introduces the classes of  $K$ -Projective and  $K$ -Injective complexes which can be used to generalize common resolutions. In this talk we will introduce the class of  $K$ -Absolutely pure complexes, we will study their basic properties and we will compare them with the class of  $K$ -Flat complexes of Spaltestein.

**Kostas Karagiannis (University of Manchester)**

**Finite group schemes acting on schemes**

The classic setup of linear representation theory of finite groups can be generalized in two directions: either by replacing the acting object, the finite group, by a finite group scheme or by replacing the object acted on, the vector space over a field, by a quasicoherent sheaf on a scheme of finite type. In this talk, I will discuss how classic problems, such as decomposing a representation into indecomposable summands or lifting an action from prime to zero characteristic, can be tackled in this generalized context. I will present past and on-going work on finite groups acting on sheaf cohomology on projective curves, and unipotent group schemes acting on arbitrary affine schemes.

**Mentzelos Melistas (Steklov Mathematical Institute)**

**Tamagawa numbers of elliptic curves with torsion points**

Let  $K$  be a global field and let  $E/K$  be an elliptic curve with a  $K$ -rational point of prime order  $p$ . In this talk, we will be concerned with how often the (global) Tamagawa number  $c(E/K)$  of  $E/K$  is divisible by the prime  $p$ . This is a natural question to consider in view of the fact that the ratio (product of the Tamagawa numbers)/—Torsion in  $E(K)$ — appears in the second part of the famous Birch and Swinnerton-Dyer Conjecture. We mainly focus on elliptic curves defined over number fields, but we also prove a result for higher-dimensional abelian varieties defined over  $\mathbb{Q}$ .

**Tasos Moulinos (Institut de Mathématiques de Toulouse)**

**Adjoining roots to ring spectra and algebraic  $K$ -theory.**

Algebraic  $K$ -theory is an invariant used to study rings, schemes, and other more generalized geometric and arithmetic objects. This invariant is universal in a precise sense and lands in the category of “spectra”. This is a higher categorical notion of an abelian group, and serves as the natural home for any and all (co)homological invariants in mathematics. One can turn  $K$ -theory on itself in a sense, and study the  $K$ -theory of ring spectra; these are monoid objects in the category of spectra which generalize rings and dga’s and can be thought of as multiplicative cohomology theories.

In this talk I broadly review the above framework, and describe ongoing work with Haldun Özgür Bayındır, where we study how to “adjoin roots” in the setting of ring spectra. I will describe the construction, and explain how it relates to adjoining roots in the classical setting of discrete rings. I will then describe preliminary consequences of the construction at the level of algebraic  $K$ -theory.

**Basileios-Dionysios Moustakas (Bar-Ilan University)**

**Counting with symmetric and quasisymmetric functions**

Symmetric functions are formal power series in infinitely many variables of bounded degree which are invariant under permutations of their variables. Quasisymmetric functions are certain formal power series which generalize the notion of symmetric functions. The latter were first considered, implicitly, in Richard Stanley’s 1972 work on  $P$ -partitions, and then implicitly in Ira Gessel’s 1984 seminal paper, where he introduced the algebra of quasisymmetric functions. Symmetric and quasisymmetric functions appear often in enumerative combinatorics, representation theory of the symmetric group, discrete geometry, algebraic geometry, graph

theory, probability theory and the theory of partially ordered sets, among other. In this talk we will make a short introduction to symmetric and quasisymmetric functions and discuss their connections with permutation enumeration, representation theory and unimodality problems in enumerative combinatorics.

**Vasiliki Petrotou (University of Ioannina)**  
**The Generic Anisotropy of Simplicial 1-Spheres**

The concept of generic anisotropy is a useful tool for proving Lefschetz properties of graded commutative algebras. In the present talk, which is based on joint work with S. A. Papadakis, we will discuss the generic anisotropy of 1-dimensional simplicial spheres over an arbitrary field, and that the determinant of the middle bilinear pairing of the generic Artinian reduction of the Stanley-Reisner ring determines the simplicial 1-sphere.

**Kostas I. Psaromiligkos (The University of Chicago)**  
**Geometry of  $p$ -adic representations**

We will construct the Lafforgue variety, a parametrizing space for the smooth irreducible representations of a  $p$ -adic reductive group  $G(F)$ . Our main tools will be Hecke algebras and a noncommutative version of the Hilbert scheme. The Lafforgue variety comes equipped with a finite-to-one projection to the Bernstein variety, which is a bijection outside the locus of a regular function that we will call discriminant, and yields important representation-theoretic information. We will give a conjectural formula for the discriminant and prove it in specific cases and explain everything concretely for the case of  $GL_2$ .

**Michail Savvas (University of Texas at Austin)**  
**Reduction of stabilizers and generalized Donaldson-Thomas invariants**

Starting with a sufficiently nice Artin stack, we explain a canonical blowup procedure that produces a Deligne-Mumford stack, resolving the locus of points with infinite automorphism group. This construction can be applied to moduli stacks parametrizing semistable sheaves on Calabi-Yau threefolds. We show that their stabilizer reductions admit natural virtual fundamental cycles, allowing us to define generalized Donaldson-Thomas invariants which act as counts of these objects. Everything in this talk is expected to be the shadow of a corresponding phenomenon in derived algebraic geometry, giving a new, derived perspective on Donaldson-Thomas theory. Based on joint works with Young-Hoon Kiem, Jun Li and Jeroen Hekking.

**Dimitra-Dionysia Stergiopoulou (National and Kapodistrian University of Athens  
& University of Thessaly)**

**Relating homomorphism spaces between Specht modules of different degrees**

Let  $K$  be an infinite field of characteristic  $p > 0$ . For a partition  $\lambda$  of  $n$ , let  $S^\lambda$  be the Specht module for the group algebra  $K\mathfrak{S}_n$  of the symmetric group  $\mathfrak{S}_n$ . These modules play an important role in the representation theory of  $\mathfrak{S}_n$ . For example, the determination of their composition factors and the corresponding multiplicities is the main open problem in the area. Relatively few results are known concerning homomorphism spaces  $\text{Hom}_{\mathfrak{S}_n}(S^\mu, S^\lambda)$  between Specht modules.

If  $\lambda = (\lambda_1, \dots, \lambda_n)$  is a partition and  $m$  a nonnegative integer, we denote by  $\lambda + (m)$  the partition  $(\lambda_1 + m, \lambda_2, \dots, \lambda_n)$ . D. Hemmer raised the question of finding general theorems that relate the spaces  $\text{Hom}_{\mathfrak{S}_n}(S^\mu, S^\lambda)$  and  $\text{Hom}_{\mathfrak{S}_{n'}}(S^{\mu^+}, S^{\lambda^+})$ , where  $n' = n + kp^d$ ,  $\lambda^+ = \lambda + (kp^d)$ ,  $\mu^+ = \mu + (kp^d)$  and  $d, k$  are positive integers. Motivation for this was the work of Henke and Henke and Koenig where, among other results, equalities of related decomposition numbers for Schur algebras of degrees  $n$  and  $n + kp^d$  for the general linear group were obtained. In this talk we will discuss the following periodicity result.

**Theorem 1.** *Let  $K$  be an infinite field of characteristic  $p > 2$ , let  $\lambda = (\lambda_1, \dots, \lambda_n)$ ,  $\mu = (\mu_1, \dots, \mu_n)$  be partitions of  $n$  and let  $k, d > 0$  be integers. If  $p^d > \min\{\lambda_2, \mu_1 - \lambda_1\}$  and  $\mu_2 \leq \lambda_1$ , then*

$$\text{Hom}_{\mathfrak{S}_n}(S^\mu, S^\lambda) \simeq \text{Hom}_{\mathfrak{S}_{n'}}(S^{\mu^+}, S^{\lambda^+}).$$

We will also discuss a related result concerning  $\text{Ext}^1$  between Weyl modules for  $GL_n$ . This is joint work with M. Maliakas.

**Christos Tatakis (University of Ioannina)**

### **Quadratic robust and generalized robust toric ideals of graphs**

A toric ideal is called robust if its universal Gröbner basis is a minimal set of generators, and is called generalized robust if its universal Gröbner basis equals its universal Markov basis (the union of all its minimal sets of binomial generators). Robust and generalized robust toric ideals are both interesting from both a Commutative Algebra and an Algebraic Statistics perspective. However, only a few nontrivial examples of such ideals are known. In this talk we study these properties for toric ideals of graphs. We characterize combinatorially the graphs giving rise to robust and to generalized robust toric ideals generated by quadratic binomials.

This is joint work with I.Garcia-Marco

**Alexios Terezakis (National and Kapodistrian University of Athens)**

### **A canonical ideal approach to the deformation theory of curves with automorphisms**

The deformation theory of curves is studied by using the canonical ideal. The problem of lifting curves with automorphisms is reduced to a lifting problem of linear representations.

**Nikolaos Tsakanikas (Saarland University)**

### **On the existence of minimal models**

The Minimal Model Program (MMP) plays a fundamental role in the classification theory of complex projective varieties up to birational equivalence. One of the central open problems of the MMP in higher dimensions is the existence of minimal models. In this talk I will discuss recent progress towards the existence of minimal models conjecture and I will also briefly explain the close relationship between the existence of minimal models and the existence of weak Zariski decompositions. This is based on joint works with Vladimir Lazić.

**Dafni Tsipa (University of the Aegean)**

**On the linearity of fundamental groups of certain graph of groups with free abelian vertex groups**

We will present results on the linearity of fundamental groups of graphs of groups. More specifically, let  $(G, \Gamma)$  be a graph of groups, with free abelian vertex groups and cyclic edge groups such that their image under the monomorphisms to the corresponding vertex groups can be extended to new generating sets. Then, we prove the linearity of the fundamental group  $\pi_1(G, \Gamma)$  for certain graphs. The main idea of the proof is to embed these groups to groups that have subgroups of finite index which are right-angled Artin groups and thus are linear. The present project is part of a PhD thesis.

**Ioannis Tsouknidas (National and Kapodistrian University of Athens)**

**Syzygies and curves**

Initiating from a major theorem of Hilbert, the theory of syzygies bridges distinct areas of mathematics under the fundamental notions of generators and relations. The work of Brill and Noether along with the Italian school of geometry brought syzygies in algebraic geometry. Koszul cohomology comes into play when one defines Betti numbers for curves and one aims to link the algebraic with the geometric setting, as conjectured by M. Green. In this talk I will elaborate on the theory and discuss open problems and recent developments.

**Laertis Vaso (NTNU)**

**Some classification results in higher-dimensional Auslander–Reiten theory.**

In higher-dimensional Auslander–Reiten theory, a central role is played by  $n$ -cluster tilting subcategories. However, there are still many unanswered questions about such subcategories. One reason for that is that finding examples is not an easy task. In this talk I will first give some background and motivation for higher-dimensional Auslander–Reiten theory and then present some classification results for some classes of algebras. If time permits, I will also explain how these classifications can help us answer some questions about this theory.

**Sokratis Zikas (University of Basel)**

**Connected algebraic subgroups of  $Bir(C \times \mathbb{P}^n)$  not contained in a maximal one.**

The classification of maximal connected algebraic subgroups of  $Bir(\mathbb{P}^m)$  for  $m = 2$  and  $3$  implies that every connected algebraic subgroup of  $Bir(\mathbb{P}^m)$  is contained in a maximal one. Thus a natural question is whether a similar statement is true for the group of birational transformations of  $C \times \mathbb{P}^n$ , where  $C$  is a curve of positive genus. In this talk, I will give a negative answer to the previous question. The proof relies on the modern machinery of the Minimal Model Program, as well as the  $G$ -equivariant Sarkisov Program. This is joint work with Pascal Fong.